Assumptions used in calculating multi-bunch growth rates and determining stability

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ASD

Random list of assumptions/points (before logical presentation)

- Ways to calculate/estimate a growth rate
  - Full tracking, modal analysis for arbitrary bunch pattern, or simple formula for equidistant bunches
- Which damping rate to use
  - Synchrotron radiation or coherent damping. Beneficial in transverse plane only
- How growth/damping from HOM occurs
- Give example of frequency dependence of growth rate
- HOM frequency is of primary importance
- Shunt impedance definition convention
  - “linac” for input power calculation, “circuit” for instability calculation
- Assumption of LOM and HOM data
- What is the accuracy of Q value prediction
  - With dampers final Q is uncertain, but is very low

Random list of assumptions/points (before logical presentation)

- Model of frequency randomness
  - Need to invent a reasonable one before a model of such can be determined with real cavities
- Model of frequency staggering
  - Staggering avoids overlap of HOMs of different cavities. Less beneficial when Qs are low.
- Bunch patterns
  - Repeat all calculations for each bunch pattern type
- Characteristics consequences of very low Q (less than 1000)
  - Single HOM can affect many possible multi-bunch modes
- How to display Monte Carlo data
  - Histogram, cumulative distribution for comparison between cases

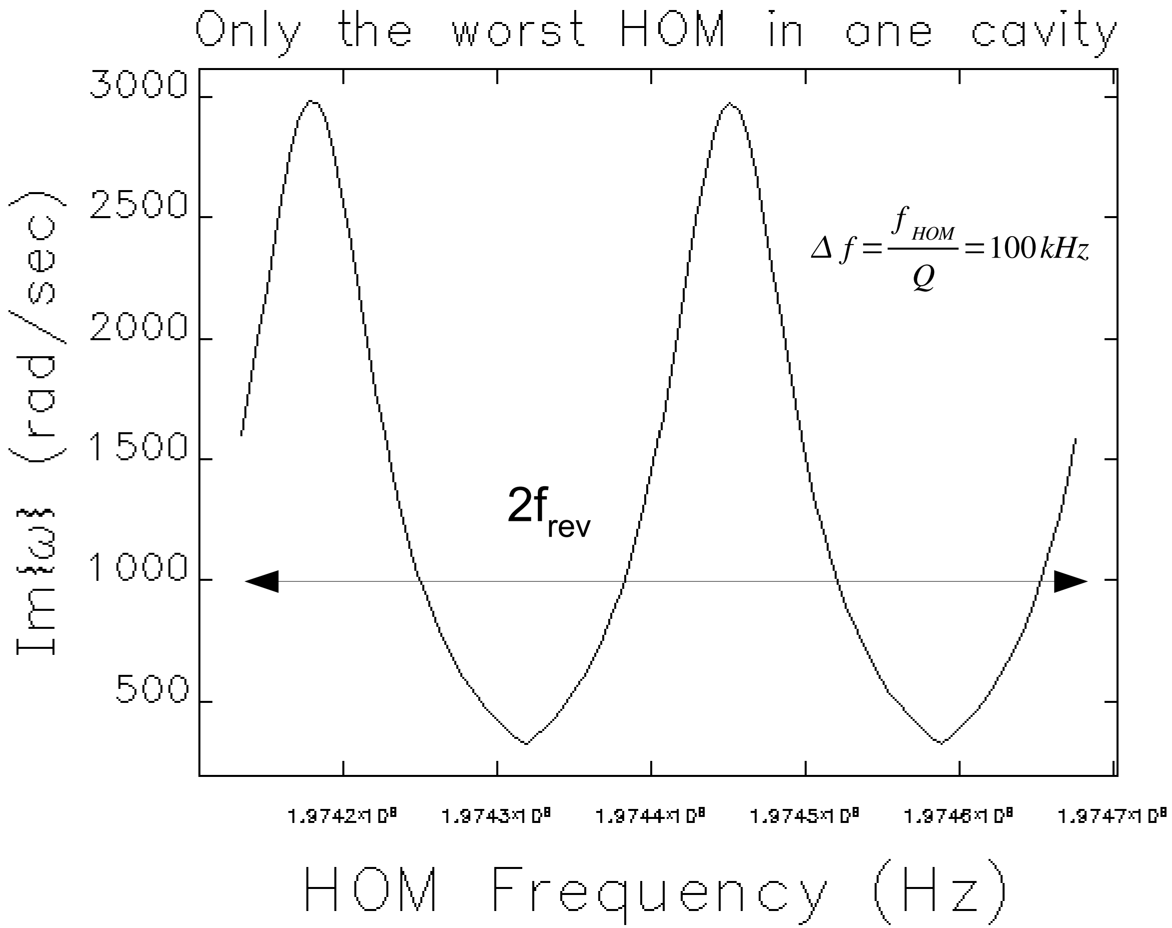
Present APS HOM and Multi-Bunch Stability Situation*

- Long range wake-fields or impedances:
  - Several monopole and dipole HOM resonators from the 16 single-cell 352-MHz cavities. Staggered HOM resonant frequencies.
  - Strong resistive-wall impedance from Al chambers (longitudinal and transverse)
- Long range wake-fields cause multi-bunch instabilities in general

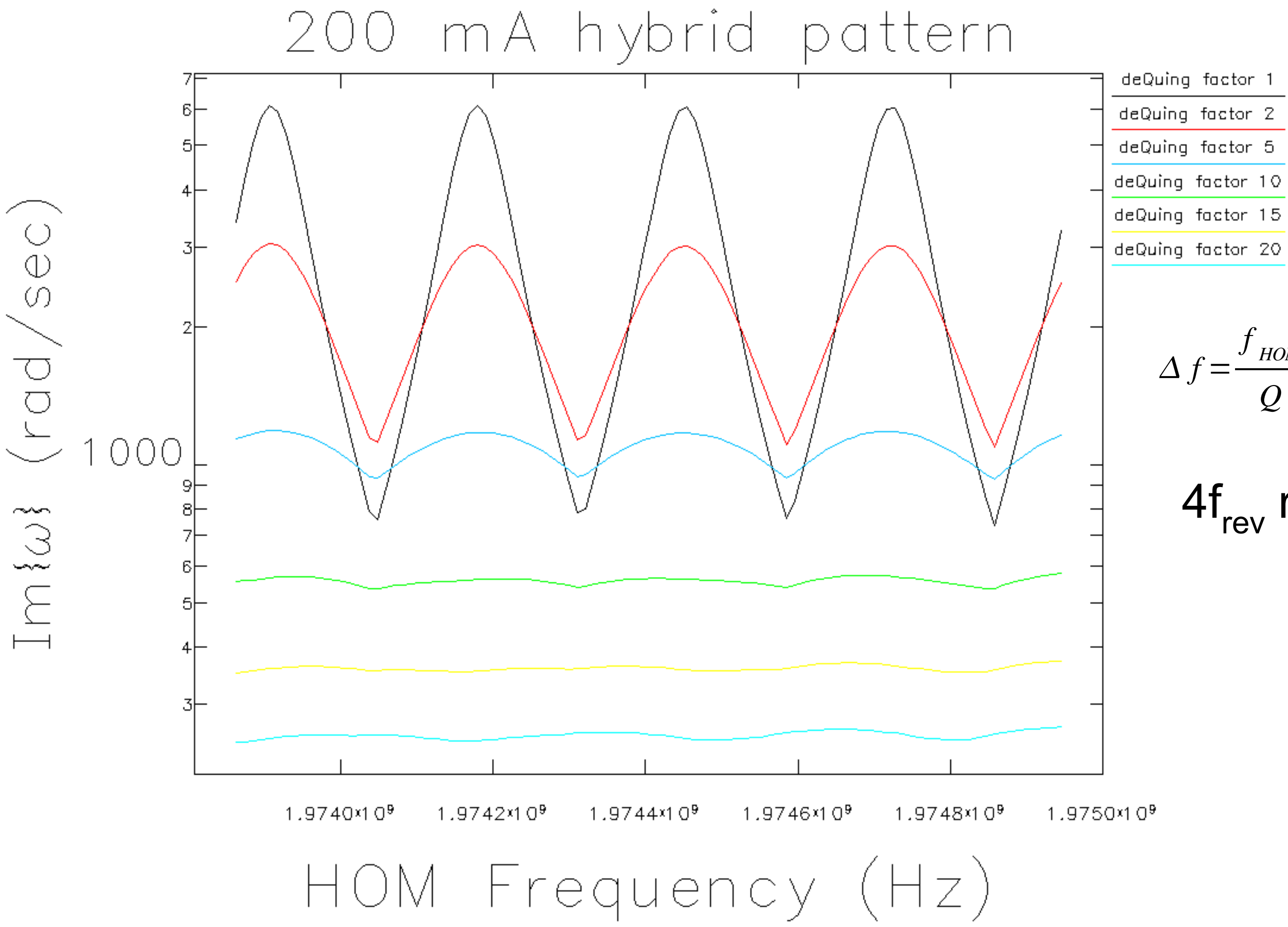
Present APS HOM and Multi-Bunch Stability Situation*

- Stability of multi-bunch modes
  - Longitudinal from rf cavity monopole HOM cured with temperature adjustments and, in the case of one group of four cavities, HOM dampers. Stable up to I = 245 mA.
  - Transverse multi-bunch instability from resistive wall cured with head-tail damping of individual bunches from positive x- and y-chromaticity
  - Rf cavity HOM dipole resonators are expected to be strongest in H-plane because of β_x/β_y ratio but coupled bunch instabilities due to them are not observed (lucky for us)

Longitudinal plane growth rate for worst Rs



Vary deQing of one mode



$$\Delta f = \frac{f_{HOM}}{Q} = 100\, kHz$$

$4f_{rev}$ range

Calculation Method[1]

- Randomized all f_{HOM} by adding a number between $-f_s$ and f_s. Use 200 samples.
- Staggering of frequencies applied to all HOM in cavities is possible; presently have none for deflecting cavities
- Use clinchor[1] based on normal mode analysis for general bunch pattern[2] for calculating worst growth rates for each case of HOM
  - Plot histogram of growth rates for each Q value
- If growth rates < damping rate, then beam is stable

[1] "Required Cavity HOM deQuing Calculated from Probability Estimated of Coupled Bunch Instabilities in the ASP Ring," L. Emery, PAC 1995

[2] "Transverse Coupled-Bunch Instabilities in Damping Rings of High-Energy Linear Colliders," K. Thompson and R. Ruth, PRD, 43, 1991

Beam Parameters

Operating total current	202 mA
Energy	7 Gev
Revolution frequency	271.55 kHz
Synchrotron frequency	2.1 kHz
Momentum compaction	2.8×10^{-4}
Cavity β_x	22 m
Cavity β_y	7.5 m
RMS bunch length for 2 mA	37 ps
Chromaticity	>6

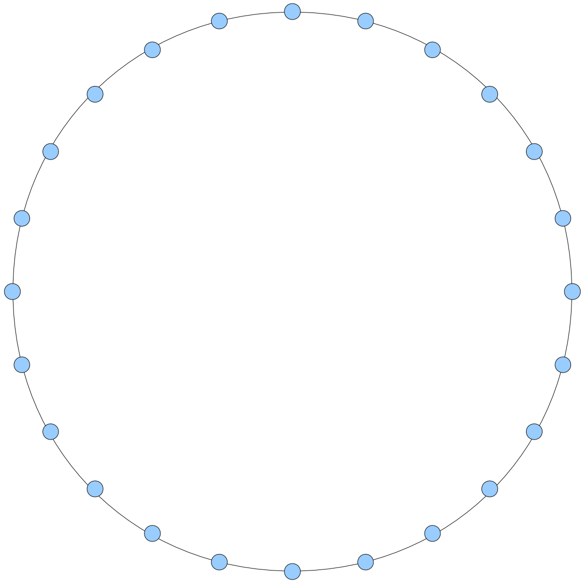
Damping rates

Planes	Synchrotron radiation	Coherent
Longitudinal	208 s^{-1}	208 s^{-1}
Horizontal	104 s^{-1}	>600 s^{-1}
Vertical	104 s^{-1}	>600 s^{-1}

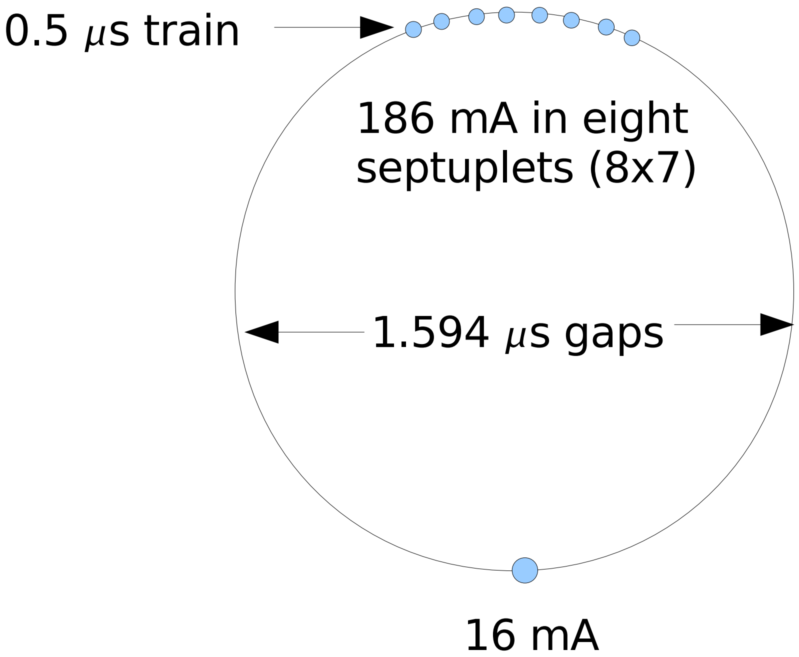
Bunch Patterns

Symmetric
202 mA in 24 bunches

Hybrid, i.e.
Single bunch plus a bunch train
202 mA in Hybrid: 1+ 8x7
Hybrid usually has higher growth rates



153 ns spacing



Growth Rate Formula for Symmetric Pattern

- Applicable for one single HOM ($\omega_r = 2\pi f_r$) at a time
- One particular beam mode will have the largest growth rate
- If the longitudinal HOM frequency is equal to a positive sideband of the beam spectrum $f_r = pNf_0 + mf_0 + f_s$ then the growth rate is maximum and is given by

$$G_s = \frac{\alpha_c I_{total}}{2(E/e)\nu_s}(R_s f_r)\ \exp(-\omega_r^2 \sigma_s^2)$$

Bunch form factor

- If the dipole HOM frequency is equal to a negative sideband of the beam spectrum for plan $f_r = pNf_0 + mf_0 - f_u$ where u=x,y then the growth rate is maximum and is given by

$$G_u = \frac{f_0 I_{total}}{2(E/e)}(\beta_u R_u)\ \exp(-\omega_r^2 \sigma_s^2)$$

Allowable Limit on Shunt Impedances for Single HOM

- Applicable only for single HOM at a time and symmetric bunches
  - Hybrid bunch pattern may be some factor worse
- Use as initial guide for damping requirements
- Table includes bunch form factor
- H-plane is worse because of ring optics functions
- Longitudinal limit assumes f_r is around 2 GHz

Plane	Shunt Impedance
Longitudinal	0.5 MΩ-GHz or 0.25 MΩ
Horizontal	1.5 MOhm/m
Vertical	4.5 MOhm/m

Shunt impedance definitions:

$$R_s = \frac{|V|^2}{2P_c} \qquad R_t = \frac{R_s(r)}{kr^2}$$

Example H-plane HOMs from SC cavity with dampers

Frequency	Q	RoverQWald	RoverQ	ShuntImpedance
Hz		Ohm/m$a2$n	Ohm/m	Ohm/m
2.96e+09	440	18.80	583	2.57e+05
3.28e+09	850	3.29	113	9.61e+04

"linac" convention

$$\left(\frac{R_t}{Q}\right)' = \frac{R_s(r)/Q}{(kr)^2}$$

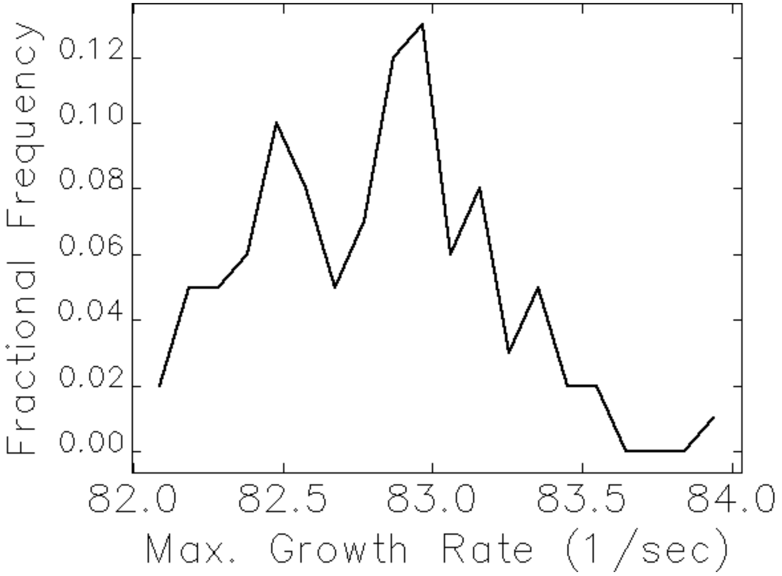
$$R_s(r) = \frac{|V|^2}{P_c}$$

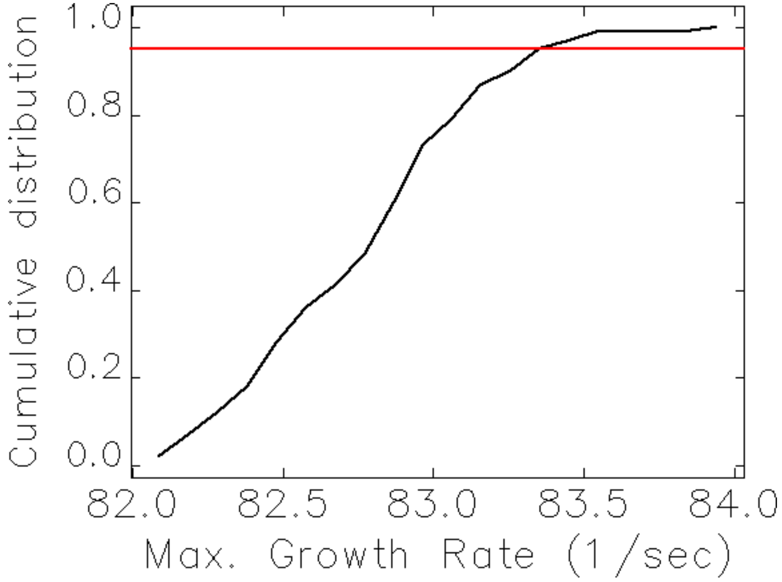
"circuit" convention

$$\frac{R_t}{Q} = \frac{R_s(r)/Q}{kr^2}$$

$$R_s(r) = \frac{|V|^2}{2P_c}$$

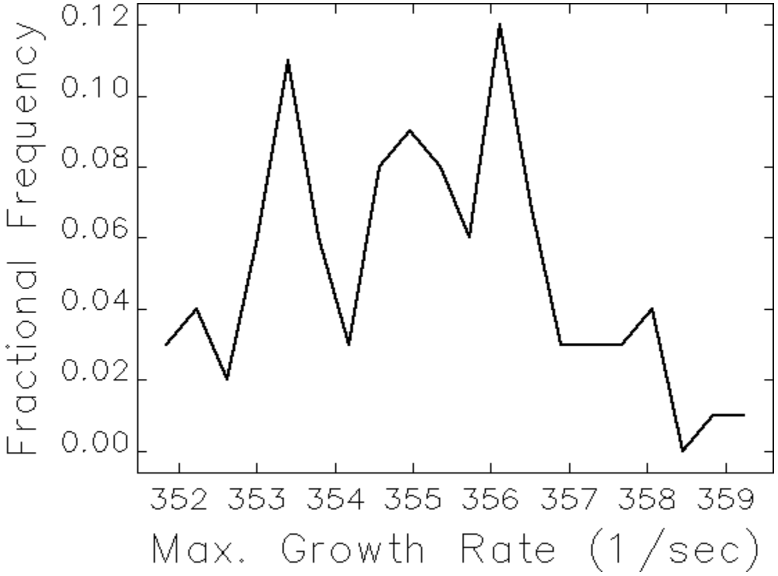
Results for H-plane

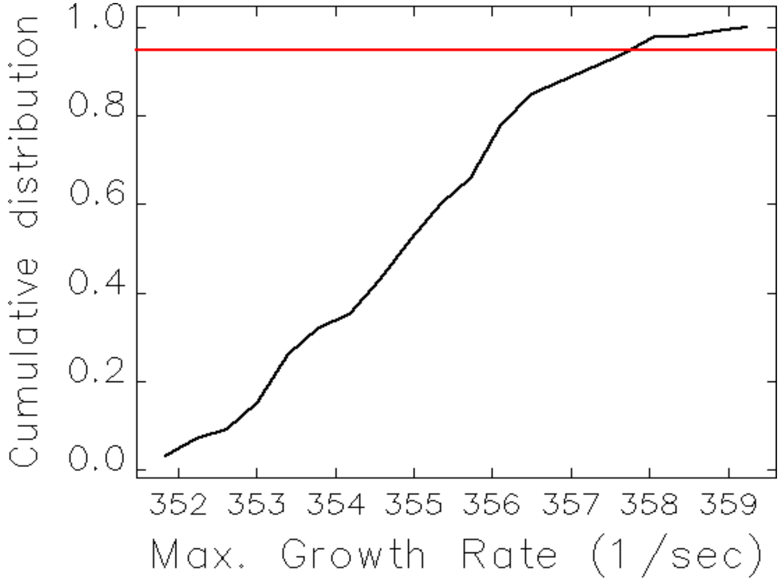




Q's as supplied by Waldschmidt

Results for H-plane – increase Q a bit





All Q's set to 1000 (about 2x for worst mode)

Growth rate increased by 4.5 !

More exploration of parameters required, i.e. staggering

Stability Result

- Q's of longitudinal and transverse planes are very low (20-200)
- Transverse plane is stable with only synchrotron radiation

Plane	Growth Rate	Damping Rate		Comment
		Synchrotron Radiation	Coherent	
Longitudinal		208 s^{-1}	Not applicable	
Horizontal	85 s^{-1}	104 s^{-1}	>600 s^{-1}	Stable
Vertical		104 s^{-1}	>600 s^{-1}	Probably stable